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Multivariable Predictive Control in the Form of State Space for a Chemical Multivariable System

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***Abstract*— The main objective in this work is to control design predictive in the form of state space for a chemical multivariable system. Our contribution is to implement this command by introducing the matrix D (input / output coupling matrix) into the control algorithms. This application will allow us to illustrate and concretize the notion of prediction with a continuous binary distillation system having state matrices D different from zero. We simulate the system for different set point changes and disturbance rejection. The obtained results have shown the best of model predictive control performances.**

***Keywords*— Distillation column, modelling, predictive control, chemical process, multivariable system**

I. Nomenclature

i : Stage number from bottom
n : Number of stages, including reboiler
N_{tot} : Number of stages, including condenser $n+1$
N_F : Feed stage
x, y : Liquid and vapour composition
y_D : Composition of light component in distillate
x_B : Composition of light component in bottom
z_F : Feed composition
q_F : Fraction of liquid in feed
M : Liquid holdup on stage i
M_D, M_B : Liquid holdup in condenser and reboiler
L : Reflux flow
V : Boil up flow
F : Feed flow rate
D : Distillate product flow rate
B : Bottom product flow rate
α : Relative volatility
τ_L : Time constant for liquid flow dynamics

II. Introduction

Distillation is a very old separation technology for separating liquid mixtures. It is today the most important technique in chemical process industry, accounting for 90 to 95% of separation operations.

According to [1], distillation is a process in which a liquid or vapour mixture of two or more substances is separated into its component fractions of desired purity, by the application and removal of heat.

Improved control of distillation can have a significant impact on reducing energy consumption, product purification.

The control of a distillation column can be difficult and that due to the feed flow rate, feed composition, from the cooling water etc.

Different strategies of control was proposed in the literatures for many years [2]: internal model control method (IMC) [3], ratio control [4,5], non-interacting control [4-6], fuzzy control [7], μ-synthesis method [8], linear-quadratic Gaussian with loop transfer recovery (LQG/LTR) method [9], Multivariable model predictive control [10], robust control [11] but the proportional Integral derivative (PID) controllers still represent the main of the controllers used in the industry.

There are many types of distillation columns based on different classifications such as: batch, continuous, binary, and multiproduct, extractive and azeotropic.

In this paper, the MPC (model predictive control) control is studied on continuous binary distillation column. However, due to the strong cross coupling and significant time delays inherent to the distillation column, the simultaneous control of overhead and bottom compositions using reflux flow and boil up flow as the control variables is still difficult.

III. Multivariable Predictive Control in The Form of State Space

The application of the MPC control law requires the presentation of the model we need to control [10].

A discretization in the form of space of state of the chemical multivariable system, it is written following form:

$$\begin{cases} x(k+1) = A \cdot x(k) + B \cdot u(k) \\ y(k) = C \cdot x(k) + D \cdot u(k) \end{cases} \quad (1)$$

Where:

$x(k) \in \Re^{n \times n}$: contains n variables model states,

$u(k) \in \Re^{m \times 1}$ $u(k) \in \Re^{m \times 1}$: m describes the model inputs,

$y(k) \in \Re^{p \times 1}$ $y(k) \in \Re^{p \times 1}$: p is the output vetor.

$k \in \mathrm{N}$: characterizes the discrete time (by definition, if T_e is the sampling period setting the continuous computer controlled system. The matrices A, B and C are the state, input and output respectely. The all-numerical state space matrices in discrete time are given in equation 26 for a sampling period $T_e = 1$ min.

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$$u(k)=u(k-1)+\Delta u(k) \tag{2}$$

In which, $\Delta u(k)$ is the control increment and the operator $\Delta = 1 - z^{-1}$ denotes the integral action which ensures static error elimination [10-12].

Taking account of the integral action equation (2), we replace $u(k)$ in the expression equation (1), for the extended condition (x_e) as follows:

$$\begin{cases} x_e(k+1) = A_e \cdot x_e(k) + B_e \cdot u(k) \\ y(k) = C_e \cdot x_e(k) + D_e \cdot u(k) \end{cases} \tag{3}$$

With the following notations:

$$x_e(k)=\begin{bmatrix} x(k) \\ u(k-1) \end{bmatrix}, \quad A_e=\begin{bmatrix} A & B \\ O_{m,n} & I_m \end{bmatrix}$$
$$B_e=\begin{bmatrix} B \\ I_m \end{bmatrix}, \quad C_e=\begin{bmatrix} C & D \end{bmatrix}, \quad D_e(k)=D \tag{4}$$

The state of the model is assumed to be available; the starting point for calculating the predicted output vector $\hat{y}(k+i|k)$ is given by the following expressions:

$$\hat{x}(k+l|k) = A \cdot x(k) + B \cdot u(k) \tag{5}$$

First, we calculate the control $u(k+i)$ iteratively from the equation (2). Predictions of future states from time k are then iteratively calculated [10]:

$$u(k+i) = u(k-1) + \sum_{l=0}^{i} \Delta u(k+l) \tag{6}$$

Predict the future states of the system, using equation (5) of the instant k to $k+i$ can be written as follows:

$$\hat{x}(k+l|k) = A^i \cdot x(k) + \sum_{j=0}^{i-1} A^{i-j-1} \cdot B \cdot u(k+j) \tag{7}$$

Finally, the future outputs of the system using equations (3), (7) of the instant k to $k+1$ can be written as follows:

$$\hat{y}(k+i) = \hat{y}(k+1|k) = CA^i x(k) + \sum_{j=0}^{i-1} CA^{i-j-1} \cdot B \underbrace{\left[u(k-1) + \sum_{l=0}^{j} \Delta u(k+l) \right]}_{u(k+j)} \tag{8}$$

The cost function J to minimize at each sampling period is to achieve the desired set-point $\hat{y}(k+i)$ following the reference trajectory $y_r(k+i)$ whose minimization provides the vector of future control $\Delta u(k+i)$ [10-13].

$$J = \sum_{i=N_1}^{N_2} \left\| \hat{y}(k+i) - yr(k+i) \right\|^2_{Q_j(i)} + \sum_{i=0}^{N_u-1} \left\| \Delta u(k+i) \right\|^2_{R_j(i)} \tag{9}$$

Here N_1 and N_2 are respectively the minimum and maximum horizon of prediction; N_u is the control horizon, $\hat{y}(k+i)$ is described in equation (9), $y_r(k+i)$ denotes the set-point at time $k+i$.

The control signals $\Delta u(k)$ result from the minimization of the following quadratic objective function with the weighting matrices Qj, Rj and the set point Yr.

The criterion to be minimized can be written in the following matrix form:

$$J = \left\| Y(k) - Yr(k) \right\|^2_{Q_j(i)} + \left\| \Delta u(k) \right\|^2_{R_j(i)} \tag{10}$$

The next step is to define the vector matrix form $\|\Delta u(k)\|$ predictions output by considering the expression (8) for $i = \overline{N_1, N_2}$ with the condition:

$\Delta u(k+i) = 0$ for $i \geq N_u$

The predicted output vector becomes [10-13]:

$$\hat{y}(k) = Y(k|k) = \Psi \cdot x(k) + \Theta \cdot u(k-1) + \Theta_\Delta \cdot \Delta u(k)$$

In which:

$$Y(k) = \left[\hat{y}(k+N_1|k), \ldots, \hat{y}(k+N_2|k) \right]^T$$
$$Yr(k) = \left[yr(k+N_1|k), \ldots, yr(k+N_2|k) \right]^T \tag{11}$$
$$\Delta u(k) = \left[\Delta u(k), \ldots, \Delta u(k+N_u-1) \right]^T$$

$$\Psi = \begin{bmatrix} C \cdot A^{N_1} \\ \vdots \\ C \cdot A^{N2} \end{bmatrix}, \Theta = \begin{bmatrix} \sum_{N1-1} \\ \vdots \\ \sum_{N_2-1} \end{bmatrix}, \sum_i = C \sum_{i=0}^{i} A \cdot e^{i-j} \cdot B \tag{12}$$

Where the state estimate $\hat{x}_e(k)$ is obtained from the observer (see Fig. 1):

$$\hat{x}(k) = A_e \cdot \hat{x}_e(k) + B_e \cdot \Delta u(k) + K(y(k) - C_e \cdot x_e(k))$$
$$\hat{x}_e(k) = (A_e - K \cdot C_e) \cdot \hat{x}_e(k) + (B_e - KD) \cdot \Delta u(k) + Ky \tag{13}$$

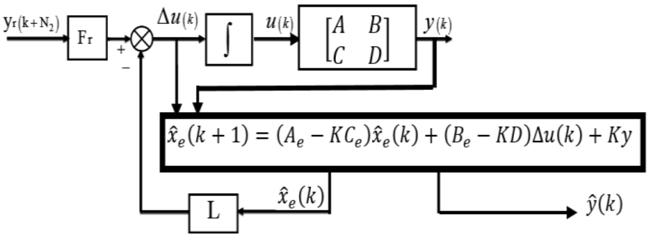

Fig. 1 The Control MPC in the form state based on the observer

The observer gain K is designed through a classical method of eigenvectors, arbitrarily placing the eigenvalues of $A_e - KC_e$ in a stable region, with the same control gain matrix L and a set-point pre-filter (F_r) different that it uses by [10-13]. As shown in Fig. 1, the control signal has the following form:

$$\Delta u(k) = F_r \cdot y_r(k+N_2) - L \cdot \hat{x}_e(k) \tag{14}$$

The matrices set-point pre-filter states are given in the form:

$$A_{Fr}(k) \in \Re^{n \times (N_2 - N_1) \times p(N_2 - N_1)}, \; B_{Fr}(k) \in \Re^{p \times (N_2 - N_1)},$$
$$C_{Fr}(k) \in \Re^{m \times p(N_2 - N_1)}, \; D_{Fr}(k) \in \Re^{m \times p}$$

$$A_{Fr}(k) = \begin{bmatrix} 0_{p(N_2-N_1) \cdot p} & I_{p(N_2-N_1)} \\ 0_p & 0_{p \cdot p(N_2-N_1-1)} \end{bmatrix}, B_{Fr}(k) = \begin{bmatrix} 0_{p(N2-N1) \cdot p} \\ I_p \end{bmatrix} \tag{15}$$

$$C_{Fr}(k) = \begin{bmatrix} 0_{p(N_2-N_1-1) \cdot p} & I_{p(N_2-N_1)} \\ 0_p & 0_{p \cdot p(N_2-N_1-1)} \end{bmatrix}, D_{Fr}(k) = \mu \cdot N_2 - N_1 + 1$$

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IV. CHEMICAL MULTIVARIABLE SYSTEM: MODEL OF THE DISTILLATION COLUMN

The aim of the distillation column is to separate the feed F, which is a mixture of a light and a heavy component with composition z_F, into a distillate product D with composition y_D, which contains most of the light component, and a bottom product B with composition x_B, which contains most of the heavy component [15].

For this objective, the column contains a series of trays that are located along its height. The liquid in the columns flows through the trays from the top to the bottom, while the vapour in the column rises from the bottom to the top. The constant contact between the vapour and liquid leads to increase the concentration of the more volatile component in the vapour, while simultaneously increase the concentration of the less volatile component in the liquid.

The operation of the column requires that some of the bottom product is reboiled at a rate of V to ensure the continuity of the vapour flow and some of the distillate is refluxed to the top tray at a rate of L to ensure the continuity of the liquid flow.

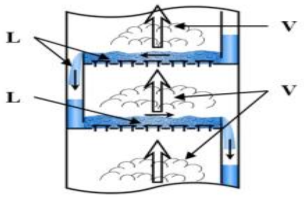

Fig. 2 A series of trays in a distillation column

In general, the distillation column is separated into three sections:

- Rectifying section: The more volatile component is removed through contacting the rising vapour with the down-flowing liquid.
- Feed section.
- Stripping section: the down-flowing liquid is stripped of the more volatile component by the rising vapour.

The nonlinear model equations are [15-16]:

$$\begin{cases} d\dfrac{M_i}{dt} = L_{i+1} - L_i + V_{i-1} - V_i \\ d\dfrac{(M_i\, x_i)}{dt} = L_{i+1}x_{i+1} - L_i x_i + V_{i-1}y_{i-1} - V_i yi \end{cases} \tag{16}$$

- That yield to:

$$d\frac{x_i}{dt} = \frac{\left(d\dfrac{(M_i\, x_i)}{dt} - x_i\left(d\dfrac{(M_i)}{dt}\right)\right)}{M_i} \tag{17}$$

- While considering some auxiliary equations

$$\begin{cases} y_i = \dfrac{\alpha x_i}{(1+(\alpha-1)x_i)} \\ V_i = V_{i-1} \\ L_i = L0_i + (M_i + M0_i)/\tau_L + \lambda(V_{i-1} - V0_{i-1}) \end{cases} \tag{18}$$

- For the feed stage, we have: $i = nf$

$$\begin{cases} d\dfrac{M_{nf}}{dt} = L_{nf+1} - L_{nf} + V_{nf-1} - V_{nf} + F \\ d\dfrac{(M_{nf}\, x_{nf})}{dt} = L_{nf+1}x_{nf+1} - L_{nf}\, x_{nf+1} + V_{nf-1}x_{nf-1} - V_{nf}\, x_{nf} + F\, z_F \end{cases} \tag{19}$$

- For the total condenser, where: $i = NT = n+1$, $(M_{N+1}=M_D, L_{N+1}=L_D, x_{N+1}=y_D)$

$$\begin{cases} d\dfrac{M_{n+1}}{dt} = V_n - L_{n+1} - D \\ d\dfrac{(M_D\, x_D)}{dt} = V_n y_n - y_D(L_D + D) \end{cases} \tag{20}$$

$M_D = C^{te}$, according to the previous auxiliary equations (18): $V_n = (L_D + D)$

- For the reboiler at $i = 1, (M_1 = M_B,\ V_1 = V_B = V, x_1 = x_B)$

$$\begin{cases} d\dfrac{M_B}{dt} = L_2 - V_1 - B \\ d\dfrac{(M_B\, x_B)}{dt} = L_2 x_2 - V_B y_1 - B x_B \end{cases} \tag{21}$$

$M_B = C^{te}$, according to equations (18): $L_2 = (L_1 + B)$

- For the second stage, we have: $i = 2$

$$\begin{cases} d\dfrac{M_2}{dt} = L_3 - L_2 + V_2 - V_1 \\ d\dfrac{(M_2\, x_2)}{dt} = L_3 x_3 - L_2 x_2 + V_1 y_1 - V_2 x_2 \end{cases} \tag{22}$$

- At Stripping section, we have: $2 \geq i \geq nf - 1$

$$\begin{cases} d\dfrac{M_i}{dt} = L_{i+1} - L_i + V_{i-1} - V_i \\ d\dfrac{(M_i\, x_i)}{dt} = L_{i+1}x_{i+1} - L_i x_i + V_{i-1}y_{i-1} - V_i yi \end{cases} \tag{23}$$

- At rectifying, we have: $nf + 1 \geq i \geq n$

$$\begin{cases} d\dfrac{M_i}{dt} = L_{i+1} - L_i + V_{i-1} - V_i \\ d\dfrac{(M_i\, x_i)}{dt} = L_{i+1}x_{i+1} - L_i x_i + V_{i-1}y_{i-1} - V_i y_i \end{cases} \tag{24}$$

- For the stage before last, we have: $i = n$

$$\begin{cases} d\dfrac{M_n}{dt} = L_{n+1} - L_n + V_{n-1} - V_n \\ d\dfrac{(M_n\, x_n)}{dt} = L_{n+1}x_{n+1} - L_n x_n + V_{n-1}y_{n-1} - V_n y_n \end{cases} \tag{25}$$

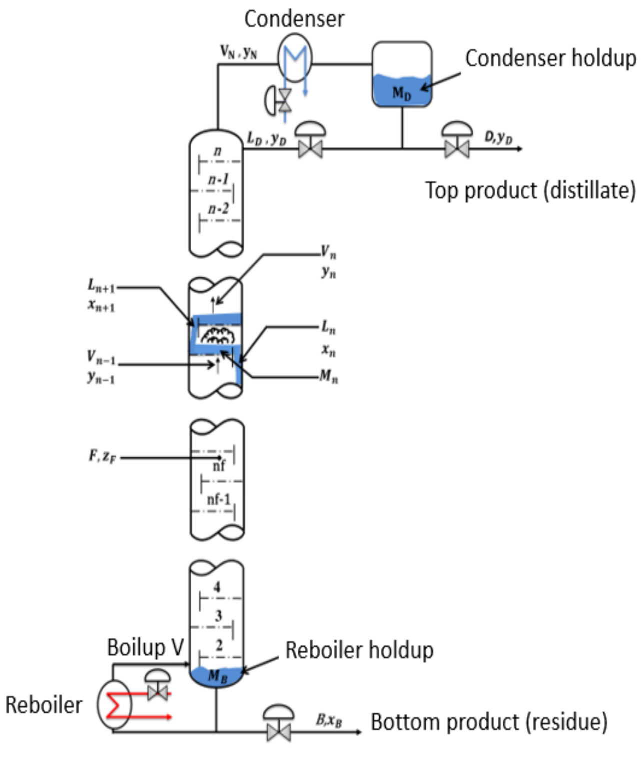

Fig. 3 Distillation unit scheme

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In this work, we use a particular high purity distillation column with 40 stages (39 trays and a reboiler) plus a total condenser is considered (Fig. 3).

There are four output variables in the distillation column:

- Composition of distillate (y_D).
- Composition of bottom product (x_B).
- Liquid holdup in condenser (M_D).
- Liquid holdup in reboiler (M_B).

The two outputs liquid holdup in (M_D and M_B) have been controlled by proportional-controllers.

There are seven input variables in the distillation column:

- Reflux flow (L).
- Boil-up flow (V).

These two manipulated variables L and V are independent for composition control.

- Distillate product flow rate (D).
- Bottom product flow rate (B).

We can use the $L-V$ configuration of stabilizing the column system, where we operation D to control M_D and B to control M_B.

- Feed rate (F).
- Feed composition (z_F).
- Fraction of liquid in feed (q_F).

These last three variables are considered as disturbances.

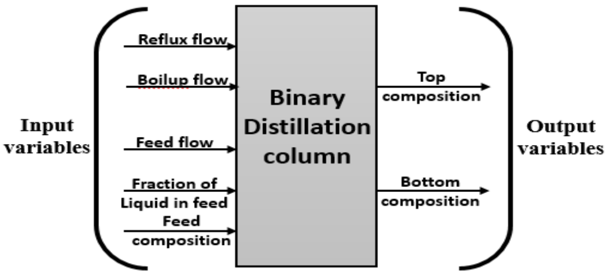

Fig. 4 Distillation process input and output variables

The matrices A, B, C and D are the state, input, output and feedforward respectively.

$$\begin{cases} A=\begin{bmatrix} 0.1673 & -0.0910 & -0.5766 & 0.1779 & 0.002 & 0.0305 \\ 0 & -0.1541 & 0.4966 & 0.2724 & -0.0012 & -0.0373 \\ 0 & -0.5586 & 0.0807 & -0.2798 & -0.0027 & 0.0828 \\ 0 & 0 & 0 & 0.7760 & 0.0011 & 0.0085 \\ 0 & 0 & 0 & 0 & 0.9948 & -0.0023 \\ 0 & 0 & 0 & 0 & 0 & 0.9274 \end{bmatrix} \\ B=\begin{bmatrix} -0.3575 & -0.1985 \\ 0.0322 & -0.0154 \\ 0.6399 & 0.1240 \\ 0.2931 & -0.1355 \\ 0.7093 & -0.6978 \\ -0.0631 & 0.2862 \end{bmatrix} \\ C=\begin{bmatrix} 0.1933 & 0.1705 & -0.0333 & 0.0273 & 0.6331 & 0.2664 \\ 1.2511 & -0.0161 & 0.1547 & 0.3080 & -0.7827 & -0.2328 \end{bmatrix} \\ D=\begin{bmatrix} 0.0124 & 0.0138 \\ 0.0790 & 0.0055 \end{bmatrix} \end{cases} \qquad (26)$$

V. Simulation Results and Discussion

In the design of the MPC controller, a difficult compromise has to be made between the set point tracking and load rejection performance.

The parameters of the MPC: $n=6, m=2, p=2$.

The weighing matrices: $Q_j = 10\text{^}9 \cdot I_p$, $R_j = I_m$.

A. Simulation of a set point change in the distillate

In this part, the set point of the composition in bottom (x_B) is fixed to 0.01 mole fraction.

The simulation was conducted for three set points:

For the first set time point $t=[0-100]$min, the set point of the composition in distillate (y_D) is equal 0.99 mole fraction. The second one, $t=[100-200]$ min, the set point of the composition in distillate (y_D) is equal 0.992 mole fraction.

The last one $t=[200-300]$ min, the set point of the composition in distillate (y_D) is equal 0.994 mole fraction.

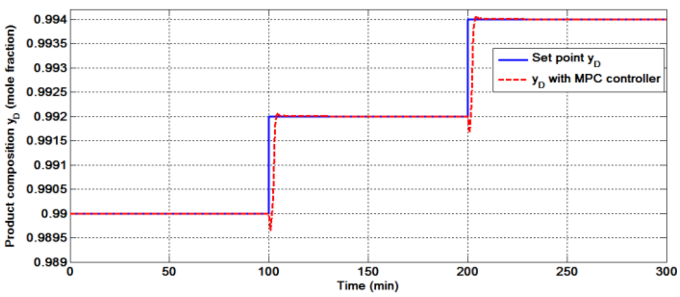

Fig. 5 Composition of light component in distillate

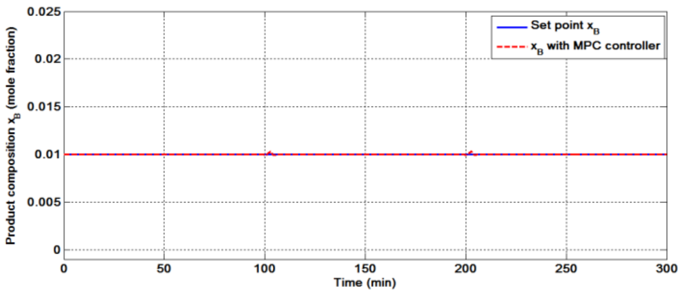

Fig. 6 Composition of light component in bottom

The Fig. 5 shows that the response of Composition of light component in distillate is satisfying in all intervals operation.

B. Simulation of a set point change in the bottom

In this part, the set point of the composition in distillate (y_D) is fixed to 0.99 mole fraction.

The simulation was conducted for three set points:

For the first set time point $t=[0-100]$min, the set point of the composition in bottom (x_B) is equal 0.01 mole fraction.

The second one, $t=[100-200]$ min, the set point of the composition in bottom (x_B) is equal 0.012 mole fraction.

The last one, $t=[200-300]$ min, the set point of the composition in bottom (x_B) is equal 0.014 mole fraction.

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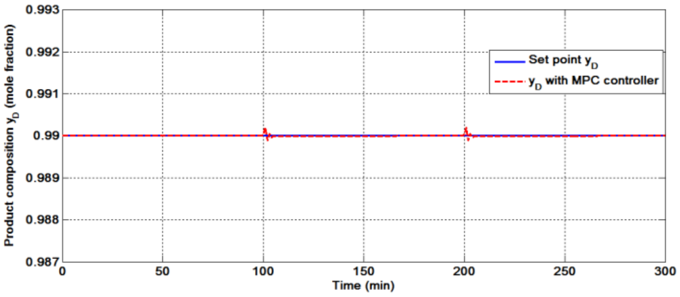

Fig. 7 Composition of light component in distillate

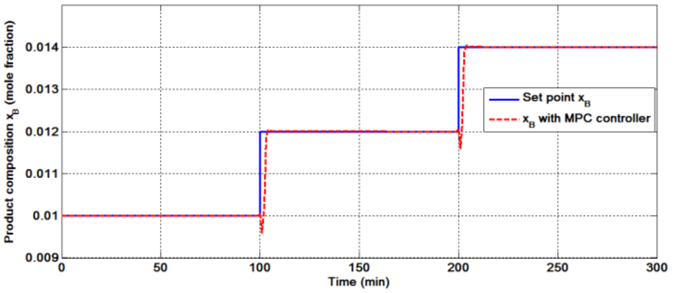

Fig. 8 Composition of light component in bottom

The Fig. 8 shows that the response of Composition of light component in bottom is satisfying in all intervals operation.

Figs. 5-8 illustrate the effect of the multivariable coupling at the instant of change of set points applied to the first (y_D) and the second set point (x_B).

The predictive multivariable controller on the distillation column shows that the interactions between control loops are eliminated (see Figs. 6-7)

C. Simulation with Disturbance Rejection

The system added a continuous disturbance step of 0.1 mole fraction after 150 min, as shown in the Figs. 9, 10.

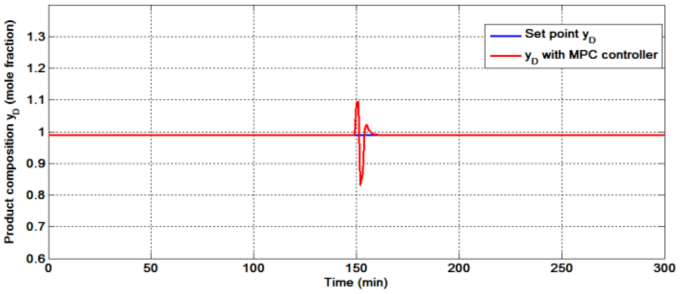

Fig. 9 Composition of light component in distillate

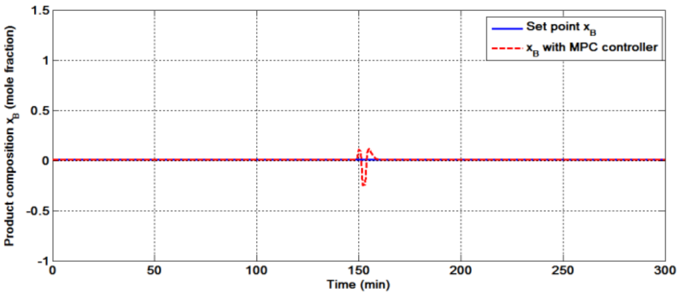

Fig. 10 Composition of light component in bottom

The MPC controller was able to reject the disturbance and has a small overshoot in the distillate and bottom compositions as shown in figs. 9, 10.

Figs. 11, 12 show the variations of the two manipulated variables: reflux flow rate (L) and steam flow rate (V) during the operation using controller to improve the system responses both for set point tracking and disturbance rejection.

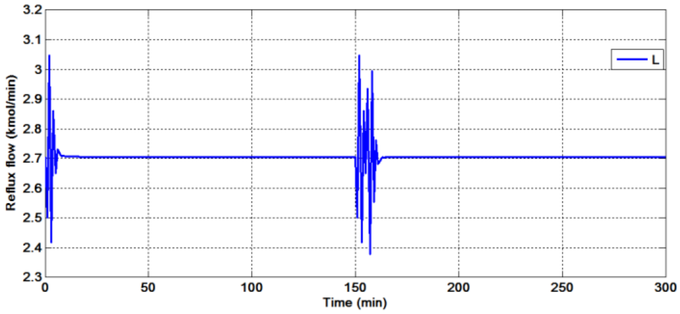

Fig. 11 Reflux flow applied with MPC controller

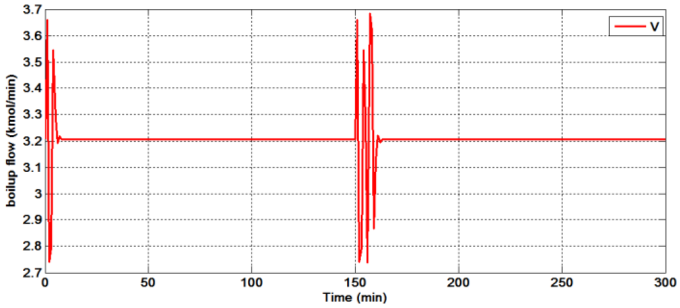

Fig. 12 Boilup flow applied with MPC controller

In Figs. 11-12, the MPC controller has an effect anticipatory on the two outputs of system with minimal future control.

VI. CONCLUSIONS

In this paper, we have used the multivariable MPC controllers to perform reference tracking and disturbance rejection in the distillation column. Changes in the discrete time observer predictive control algorithm have been made in such a way as to take into account that the matrix D is nonzero of the multivariate system of the distillation column.

We show that the two desired set points are exactly satisfied in the presence of disturbance, interaction and time delay, which demonstrates the effectiveness of the controller.

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TABLE I
UNITS FOR COLUMN DATA

Symbol	Quantity	Units
n	40	
N_{tot}	41	
n_F	21	
F	1	(kmol/min)
z_F	0.5	(mole fraction)
D	0.5	(kmol/min)
B	0.5	(kmol/min)
L	2.70629	(kmol/min)
V	3.20629	(kmol/min)
α	1.5	
y_D	0.99	(mole fraction)
x_B	0.01	(mole fraction)
M_i	0.5	(kmol)
τ_L	0.063	(min)

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